University of International Business and Economics
International Summer School

MAT 230 Multivariable Calculus (Calculus III)

Term: June 13th – July 14th, 2022
Instructor: Dr. Sergei V. Shabanov
Home Institution: University of Florida, Gainesville, USA
Email: shabanov@ufl.edu
Class Hours: Monday through Friday, 120 minutes each day (2,400 minutes in total)
Discussion sessions: Zoom video meetings, time TBA
Office hours: to be announced

Total Contact Hours: 64 contact hours (45 minutes each, 48 hours in total)
Location: WEB
Credit: 4 units

Course Description：

The course covers the following concepts: vector algebra, lines, planes, curves, and surfaces in space, functions of several variables, multivariable limits and continuity, partial derivatives and differentiation of functions of several variables, extreme values of functions of several variables and the method of Lagrange multipliers, double and triple integrals, change of variables in multiple integrals, line and surface integrals, and applications of differentiation and multiple integration to vector fields (line and surface (flux) integrals of vector fields, fundamental theorem for line integrals, etc.).

Course Goals：

1. Have facility with the basic theory and techniques of integral and differential vector calculus: e.g., the various types of vector products, notions of arc length and curvature, generalizations of the derivative (partial derivatives, directional directives, etc.); integrals of multivariable functions, change of variables, vector fields, line integrals, divergence, gradient and curl, integrals of vector fields over surfaces, etc.
2. Have precise knowledge of the definitions, theorems, and derivations from the basic theory of multivariable calculus: e.g., the geometric interpretation of the dot product, various formulae for the arc length, the relationship between gradient and directional derivatives, the change of variable formula, and various generalizations of the fundamental theorem of calculus.
3. Have facility with basic calculational skills: e.g., facility with vectors, evaluation of arc length and curvature, ability to determine tangent planes, facility with the Lagrange multiplier method, ability to calculate double and triple integrals, surface integrals, etc.

4. Have a rudimentary ability to explain mathematical theory using rigorous mathematical reasoning.

Required Textbook:

S.V. Shabanov, Concepts in Calculus III. Multivariable Calculus,
ISBN 978-1-61610-162-6, Edition of 2019.

Prerequisites:

The course is based on Calculus II or its equivalent.

Grading Policy:

Each assignment is graded out 100 points (it is possible to get extra point, in addition to 100, by solving an extra credit problem, if any offered). The average of the two exams makes 60% of the grade, and the average of the three homework assignments makes 40% of the grade. The course score is computed by the rule

G=0.6 (E1+E2)/2 +0.4(HW1+HW2+HW3)/3

where E1 and E2 are the exam score, and HW1,2,3 are the homework assignment scores.

Grading Scale:

A	90-100	**C+**	72-74
A-	85-89	**C**	68-71
B+	82-84	**C-**	64-67
B	78-81	**D**	60-63
B-	75-77	**F**	below 60

It should be noted that in many US colleges **C-** is not a passing grade if the course is required for a major.

Lectures:

All lectures are prerecorded video lectures. The lectures cannot be downloaded and will be available for multiple viewing to all enrolled students. Students are expected to view 2-3 lectures per day as directed by the syllabus and/or instructed during the course. Each lecture is about one hour long. Each video lecture corresponds one section in the textbook. In the beginning of each week students will receive an instruction which video lectures they have to study for the current week.

Q&A Zoom sessions:

Every week there will be a 2-hour Q&A zoom discussion session devoted to the material of the lectures for a current week. An instructor can answer any questions about the current week lectures, clarify some of the concepts with examples from homework assignments if necessary (or asked).

Exams:

There will be three graded homework assignments, one midterm exam, and a final exam. **The homework assignments** are not cumulative. Each assignment will contain problems from a particular part of the course. Each assignment will be open to start at a specific time and will be closed after two hours. During this designated period of time students have to solve the problems, scan their work into a PDF format, and submit to the course TA as instructed in the assignment sheet. It must be noted that that no late submission will be accepted. Each submission must contain the signed student honesty pledge (provided in the assignment sheet. A full credit for a solution is awarded ONLY if all technical details are included into the submission. NO credit for plain answers without calculations and/or reasoning. The instructor reserves a right to video interview students whose solutions have similarities that are unlikely to occur (e.g., identical small technical errors, and similar). If any of these students show the lack of ability to solve similar problems during the interview, all students in the group receive no points for the assignment (no investigation who copied from who will be conducted). By signing the honesty pledge, a student agrees to a possible video zoom interview if a violation of the academic honesty is suspected. So, it is STRONGLY recommended that the students do not discuss the assignments problems during the two-hour period after the assignment is open. Preliminary the homework assignments are scheduled on Mondays June 20, June 27, and Friday, July 1, time to be announced. **The mid-term and final exams** are cumulative. They will be conducted in class, proctored by UIBE TAs and local professors, and supervised by the instructor via zoom. The problems are released by the instructor at the beginning of the exam via zoom. Students will have 2 hours to finish the work, scan their work in front of the zoom camera, and email it to the course TA and the instructor within specified period of time. Any submission received outside the specified time window will NOT be accepted. The exams are free-response assignments. No credit for plain answers. The logic and technical details of solutions must be given in order to get a credit. No notes, no books, no calculators or any other electronic devices are permitted on the exams. One formula sheet is permitted (an A4 sheet, back and front). Makeups for missed exams are only with a written medical excuse approved by the school administration. The mid-term exam is scheduled on the third week of the school, and the final will take place on July 14.

Course Schedule: (tentative)

WEEK ONE: Vector algebra, lines and planes in space

Mon: Sections 1-3. Euclidean geometry. Lines and planes in space. Rectangular coordinate systems. Rigid transformations in space. Rotations and translations of a coordinate system. Distance between two points. Algebraic description of sets in space. Spheres and balls. Vectors. Vector algebra. Parallelogram rule. The dot product. Geometrical significance of the dot product.
Assignment: 1.10 (1, 6, 7, 9, 11, 19, 21), 2.5 (3, 4, 8, 9, 11), 3.8 (2, 4, 5, 9, 11, 15),

Tues: Sections 4-5. The cross product. Geometrical significance of the cross product. Area of a parallelogram. Area of a triangle. Criterion for two vectors being parallel. Triple product. Its geometrical significance. Volume of parallelepiped. Criterion for three vectors being coplanar.
Assignment: 4.6 (2, 3, 4, 11, 16, 19, 26), 5.5 (2, 4, 6, 8, 9, 13)
Wed: Sections 6-7. Algebraic description of lines in space. Vector, parametric, and symmetric equations of a line. Distance between a line and a point. Algebraic description of planes in space. Distance between a plane and a point.
Assignment: 6.5 (1, 3, 7, 9, 11, 21), 7.5 (2, 7, 9, 10, 11, 12, 18, 23).
Thurs: Q&A zoom session
Fri: Sections 9-10. Quadric surfaces in space. Classification of quadric surfaces. Paraboloids, hyperboloids, cones, and ellipsoids. Vector functions. Continuous vectors functions and curves in space. Parametric curves and parameterization of a space curve. Graphing parametric curves.
Assignment: 9.6 (1-5, 11-15, 25, 26). 10.4 (1-10).

WEEK TWO: Curves in space and vector functions

Mon: Homework 1 Sections 11-12. Derivative of a vector function. Its geometric significance. Tangent line to a curve. Smooth curves. Integration of vector functions. Fundamental theorem of calculus for vector functions. Reconstruction of a vector from its derivative. Solving Newton's equations in mechanics.
Assignment: 11.4 (1-3, 6, 7, 11, 19), 12.5 (1-3, 7-10, 12)
Tues: Sections 13-14. Arc length of a curve. Theorem about arc length of a smooth curve. Curvature of a smooth curve. Geometric significance of curvature. Curvature radius. Unit tangent and normal vectors. Osculating plane and circle. Applications to mechanics: tangent and normal accelerations.
Assignment: 13.3(1, 3, 4, 8-9, 16, 17), 14.3 (1-7, 14-17)
Wed: Sections 16-19. Functions of several variables. Range and domain. Graph of a function of two variables as a surface in space. Level sets. Level surfaces of a function of three variables. Limits and continuity. Limit along a curve. Squeeze principle. General strategy to analyze multi-variable limits. Continuity of polynomials. Continuity of composition of continuous functions. . Partial derivatives. Geometrical significance of partial derivatives. Partial derivatives as functions.
Assignment: 16.4 (4-12, 18-21), 17.3 (1, 2, 8, 9, 23, 24), 18.6 (4-8, 15, 16), 19.3 (3-6, 8, 12, 17).
Thurs: Q&A zoom session
Fri: Midterm Exam

WEEK THREE: Differentiation of functions of several variables

Mon: Sections 20-22. Higher order partial derivatives. Clairaut's theorem. Reconstruction of a function from its partial derivatives. Integrability conditions. Differentiability of a function as the existence of a good linear approximation. Tangent plane approximation for functions of two-variables. Linearization of a function near a point. Continuity of partial derivatives as a sufficient condition for differentiability. Chain rules and implicit differentiation. Implicit function theorem.

Assignment: 20.4 (2-6, 10, 14, 20), 21.6 (1, 5, 7-10, 14, 15, 23), 22.7(2, 6, 7, 11, 25, 26, 31)
Tues: Sections 23-24. Differential. Its geometric significance. Higher order differentials and Taylor polynomials. Accuracy of a linear approximation. Taylor polynomial approximations. Estimation of error of the Taylor approximation for functions with continuous partial derivatives. Directional derivative. Its geometric significance. The gradient and its geometric significance.
Assignment: 23.7 (1-3, 9, 10, 27, 28), 24.5(4-6, 9, 11, 15, 24, 28, 29, 45)
Wed: Sections 25- 26. Extreme values. Critical points and the gradient. Second derivative test for functions of several variables. Extreme values on a set. Extreme value theorem.
Assignment: 25.6(1, 6-10, 13-15), 26.6(1-3, 27-29).
Thurs: Q&A zoom session
Fri: Sections 27-29. Extreme values of a function of several variables on smooth curves and surfaces. The Lagrange multiplier method. The volume problem. Lower and upper sums to estimate a volume under a surface. The double integral and its properties.
Assignment: 27.6(3-6, 9, 11), 28.6(2, 10, 11, 15, 16), 29.1(3, 4, 8-10)

WEEK FOUR: Multiple integrals

Mon: Sections 30-31. Iterated integrals. Fubini's theorem. Evaluation of double integrals over general regions. Vertically and horizontally simple planar regions and their algebraic description. Reversing the order of integration in an iterated integral.
Assignment: 30.3(1-8), 31.6(1-3, 6-13, 28-33)
Tues: Sections 32-33. Reversible transformations in a plane. Change of variables in a plane. Jacobian. Change of variables in double integrals. Example: Double integral in polar coordinates. Symmetry and area preserving transformations in double integrals. The mass problem. Definition of a triple integral. Properties of a triple integral.
Assignment: 32.5(1-3, 9-11, 17-19, 27, 28), 33.5(11-13, 19, 21-23)
Wed: Sections 34-36. Fubini's theorem for triple integrals. Evaluation of triple integral over vertically simple spatial regions. Reversible transformations in space. Change of variables. Jacobian. Change of variables in triple integrals. Example: Triple integrals in spherical and cylindrical coordinates. Volume of an ellipsoid.
Assignment: 34.6(1, 2, 4, 7, 8, 13, 18, 21-23), 35.6(4-6, 9-11, 18-27)
Thurs: Q&A zoom session
Fri: Sections 38-39. Line integral along a smooth curve. Surface integral over a smooth surface. Surface area. Evaluation of the surface integral over a graph of function of two variables.
Assignment: 38.3(1, 3, 5, 14, 18), 39.5(2-4, 6, 7, 9, 28).

WEEK FIVE: Vector fields and vector calculus

Mon: Sections 41-42. Fluid and gas flows. Velocity of the flows. Vector fields. Graphic representation of a vector field. Flow lines. Line integral of a vector field over a smooth curve. Application: Work done by a force field in moving an object along a smooth curve. Conservative vector fields. Fundamental theorem for line integrals of conservative vector fields. Path-independence of line integrals and the curl of a vector field.

Assignment: 41.6(1-3, 7, 11, 17-19), 42.6(9-12, 17, 18)
Tues: Sections 44-45. Flux of a vector field across a smooth surface. Orientable surfaces. Reduction of the flux integral to a double integral. Stokes' theorem. Induced orientation of a surface by an orientation of its boundary. Reduction of a line integral over a closed path to a flux integral.
Assignment: 44.5 (1-5, 8-11, 15), 45.7 (4-8)
Wed: Sections 43, 46. Two-dimensional version of Stokes'. Green's theorem. Area of a planar region bounded by a smooth closed curve via the line integral over the curve. Flux of a vector field across a smooth orientable closed surface. Divergence (Gauss-Ostrogradsky) theorem. Divergence of a vector field. Geometric significance of the divergence and the curl of vector fields with an example of flow lines of a fluid. Sink and faucet type of sources of a vector field.
Assignment: 43.5(4-10), 46.6 (18-25)
Thurs: Q&A zoom session
Fri: Final exam