University of International Business and Economics
International Summer School

MAT 230 Multivariable Calculus (Calculus III)

Term: October 10th –December 2nd, 2022
Instructor: Shen Fan
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Email: fans@cup.edu.cn
Class Hours: 240–360 minutes each week (2,400 minutes in total)
Office hours: TBD
Discussion sessions: 60–120 minutes each week

Total Contact Hours: 64 contact hours (45 minutes each, 48 hours in total)
Location: WEB
Credit: 4 units

Course Description：

This course extends the theory of differential and integral calculus to functions of many variables. Students begin the course by covering vector operators such as dot and cross products, as well as sketching common quadric surfaces. They then move on to vector valued functions, space curves and derivatives and integrals of vector valued functions. The course then proceeds to what could be considered the Differential Calculus of Multivariable Functions. Students explore the ideas of Limit, Continuity, Partial Derivatives, Tangent Planes, Chain Rule. After learning these fundamentals, students use Directional Derivatives and Gradients to optimize functions. Lastly, students learn Lagrange Multipliers as a potentially easier and more elegant method for finding maxima and minima. Next, students move on to the Integral Calculus of Multivariable Functions. Students learn Double and Triple Integrals, as well as method such as integrating with Spherical, Cylindrical, and Polar Coordinates. Students then move on to the concepts of Vector Calculus. They are introduced to Vector Fields, Line Integrals and the Fundamental Theorem of Line Integrals. More advanced topics including Green's Theorem, Curl and Divergence, Parametric Surfaces, Surface Integrals, Stokes' Theorem, and the Divergence Theorem conclude the course.

Course Goals:

After completing this course students should be able to:

1. Have facility with the basic theory and techniques of integral and differential vector calculus: e.g., the various types of vector products, notions of arc length and curvature, generalizations of the derivative (partial derivatives, directional directives, etc.); integrals of multivariable functions, change of variables, vector fields, line integrals, divergence, gradient and curl, integrals of vector fields over surfaces, etc.

2. Have precise knowledge of the definitions, theorems, and derivations from the basic theory of multivariable calculus: e.g., the geometric interpretation of the dot product, various formulae for the arc length, the relationship between gradient and directional derivatives, the change of variable formula, and various generalizations of the fundamental theorem of calculus.
3. Have facility with basic calculational skills: e.g., facility with vectors, evaluation of arc length and curvature, ability to determine tangent planes, facility with the Lagrange multiplier method, ability to calculate double and triple integrals, surface integrals, etc.
4. Have a rudimentary ability to explain mathematical theory using rigorous mathematical reasoning.

Prerequisites:

The course is based on Calculus 1 and 2 (or their equivalents). Students are expected to know basic concepts of calculus for functions of a single real variable. Good technical skills in differentiation and integration are necessary. Particular topics of Calculus 2, such as numerical series, power series, planar curves, are not mandatory for the course. However, a basic knowledge of these topics will be very helpful as the course contains higher dimensional versions of them.

Required Textbook:

Stewart, James. Multivariable Calculus, 7th edition

Grading Policy:

The final grades will be calculated using the following percentage breakdown:

- Five Assignments: 30% total, each worth 6%.

  Marks for each homework assignment will be awarded based on the correctness of the solutions as well as:

  - ✓ **completeness** A serious effort was made at providing solutions to all the assigned problems.
  - ✓ **neatness** Solutions (and your name) are clearly and neatly written on the paper.
  - ✓ **solution addresses the problem** All that was asked for in the problem statement is provided in the solution. For problems that call for a specific computation or example (as opposed to an explanation) the answers are clearly written and boldly circled.
  - ✓ **style** Problems that require an explanation or justification are answered with a clear and logical argument written in proper sentences. Additionally, mathematical terminology is used correctly

  If you do not do an assignment, then you will receive a grade of 0 for it. No exceptions. Late work handed in is discounted 20 % per day late.

Every assignment has a hard deadline, usually 2 days past the original due date. Late submissions (penalty or not) are not accepted after the hard deadline.
No late submissions (penalty or not) will be accepted for the last assignment.

- Midterm: 30%
  There will be one 2-hour midterm in this class, it will take place on **Nov 11**.
  The midterm will cover chapters 12-14.
  It will have 8-10 computational problems, generally with several parts.
  The midterm will be closed-book. No notes, calculators, or other electronic devices will be allowed.
  There will be NO MAKE UP MIDTERMS.

- Final Exam: 40%
  The final exam is closed book and 2 hours in duration, it will take place on **Dec 2nd**.
  It will have 8-10 computational problems and short proof type questions.
  The final will be closed-book. No notes, calculators, or other electronic devices will be allowed.
  The Final Exam will be cumulative but will emphasize the material from Chapters 15-16.
  There will be NO MAKE UP FINAL EXAMS.

Grading Scale:

Assignments and examinations will be graded according to the following grade scale:

A	90-100	**C+**	72-74
A-	85-89	**C**	68-71
B+	82-84	**C-**	64-67
B	78-81	**D**	60-63
B-	75-77	**F**	below 60

It should be noted that in many US colleges **C-** is not a passing grade if the course is required for a major.

Attendance:

Summer school is very intense and to be successful, students need to attend *every class*. Occasionally, due to illness or other unavoidable circumstance, a student may need to miss a class. A medical certificate is required to be excused. Any absence may impact on the student's grade. Arriving late or leaving early will count as a partial absence. If a student is missing less than a point for a better grade, the better grade will be given, provided the student had no unexcused absences during the course.

Policies on Academic Integrity

- Collaboration on Assignments
  Collaboration with your classmates on the homework is encouraged! However, collaboration does not mean copying someone else's answers. It means working together so understands and solves the problems. You must write your own solutions in your own words.
- Cheating on exams
  Academic integrity will be strongly enforced in this course. Any student caught cheating on the exams will be given an F grade for the course and will be reported to the Office of the Dean of Students.

Course Schedule: (tentative)

Days	Topics
Oct.10	Three-dimensional Coordinates.
	Vectors.
Oct.11	Dot Product.
	Cross product.
Oct.12	Lines and Planes.
	Cylinders and Quadric Surfaces.
Oct.17	Vector Valued Functions and Space Curves; Derivatives and Integrals of Vector Functions.
	Derivatives and Integrals of Vector Functions; Arc Length.
Oct.18	Arc Length and Curvature.
Oct.24	Arc Length and Curvature.
	Functions of Several Variables.
Oct.25	Limits and Continuity.
	Partial Derivatives.
Oct.26	Partial Derivatives.
	Tangent Planes Differentiability and Chain Rule.
Oct.31	Chain Rule.
	Gradients, Directional Derivatives.

Nov.1	Max-Min Problems.
Nov.7	Lagrange Multipliers.
	Double integrals intro.
Nov.8	Iterated Integrals.
	Double Integrals.
Nov.9	Double Integrals Continued.
	Double Integrals in Polar Coordinates.
Nov.14	Triple Integrals.
	Triple Integrals in Cylindrical Coordinates.
Nov.15	Triple Integrals in Spherical Coordinates.
Nov.21	Change of Variable Formula.
	Vector Fields.
Nov.22	Line Integrals.
	Fundamental Theorem of Calculus for Line Integrals.
Nov.23	Green's Theorem.
	Div, Grad, Curl.
Nov.28	Parametric Surfaces. Surface Integrals.
	Surface Integrals, continued.
Nov. 29	Stokes's Theorem.